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Set–Reset latch logical operation induced by colored noise

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A R T I C L E I N F O

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A B S T R A C T

We examine the possibility of obtaining Set–Reset latch logical operation in a symmetric bistable system subjected to OU noise. Three major results are presented. First, we prove the Set–Reset latch logical operation can be obtained driven by OU noise. Second, while increasing the correlation time, the optimal noise band shifts to higher level and becomes wider. Meanwhile, peak performance degrades from 100% accuracy, but the system can still perform reliable logical operation. Third, at fixed noise intensity, the success probability evolves non-monotonically as correlation time increases. The study might provide development of the new paradigm of memory device.

1. Introduction

With the continuous shrinking of electronic components and the advent of new computing paradigms like molecular or DNA computing systems, the smaller power supply has brought with it problems of smaller noise margins and higher error rates. Meanwhile, the unavoidable thermal noise in electronic devices threatens to break Moore's law, and becomes a serious issue of future miniaturization with high density integration technology [1]. Several researchers have pay attention to the potential role of noise in electronic devices, trying to find how to promote success computing probability coexisting with noise and optimize future design. Gammaitoni [2] showed that the presence of noise set a fundamental limit to the computing speed. Kish [3] proposed the concept of thermal noise driven computing, where a potential application of thermal noise was exhibited by the problem of reducing power dissipation in microprocessors. However, one of the three factors of speed, error and heat should be given up because of the miniature of microelectronics, which is accompanied by the increase of thermal noise [1]. Therefore, major breakthroughs are still needed.

Noise however is not always detrimental. Benzi et al. [4] studied the phenomenon of the apparent synchrony between glacial periods and the variations of solar energy flux, and put forward the concept of stochastic resonance, wherein the addition of moderate noise induces the response of nonlinear system to enhance a weak periodic input. Since the concept of SR is proposed, it has received considerable attention in the past decades.

Recently, Murali et al. [5] introduced the concept of logical stochastic resonance (LSR), namely the phenomenon where a nonlinear system driven by weak signals representing logic inputs under optimal noise can obtain logic outputs, which is an application of SR to logic computation. Another research of Murali et al. [6] demonstrated LSR via a circuit implementation using a linear resistor, a linear capacitor and four COMS-transistors. Although LSR is a very recent idea, the number of studies on LSR is growing fast. For instance, Animesh et al. [7] found that dynamical behavior equivalent to LSR can also be obtained without noise, subjected only to an appropriate window of frequency and amplitude of the periodic influence, in a bistable system. Remo et al. [8] extended the study of LSR from the context of bistable system to multi-stable (tri-stable) system given by piecewise function and obtained XOR logic. H. Zhang et al. [9] investigated that the LSR phenomenon in a class of 3-well systems can be successfully induced by additive or multiplicative Gaussian colored noise, and obtained the approximate Fokker–Planck equation by using decoupling approximation. L. Zhang et al. [10,11] investigated the effects of different kinds of colored noise on LSR and found the moderate autocorrelation time for optimal performance. Das et al. [12] investigated the dichotomous noise induced LSR in energetic and entropic systems. Dari et al. [13] and Hellen et al. [14] studied the LSR phenomenon in synthetic gene network. Studies about LSR in chemical, nanomechanical, optical and biological systems have been investigated by Sinha et al. [15], Guerra et al. [16], Zamora-Munt [17] and Ando et al. [18].

In a recent work, Kohar et al. [19] investigated the possibility of utilizing a noisy nonlinear system, not just as a logic gate, but also directly as a memory device, using LSR concept. Inspired by Kohar et al.'s article, we focus our attention on the role of colored noise in the memory device, specifically Set–Reset latch. A latch

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Table 1
Relationship between the two inputs and the output of Set–Reset latch.

Set (I_1)	Reset (I_2)	Q_{next}	Action
0	0	Q	Hold state
0	1	0	Reset
1	0	1	Set
1	1	X	Not allowed

is a system that has two stable states and can be used to store state information. The system can be made to change state by signals applied to one or more control inputs, Set–Reset latch logical function is shown in the truth table (Table 1). The conventional latch can be built by a pair of cross-coupling inverting elements. In present computing systems, the memory and modification time are significant bottleneck limiting the speed of computation.

In this paper, we demonstrate that the Set–Reset latch operation can be obtained in symmetric bistable nonlinear system driven by moderate noise, specifically OU noise. The reliability of the logic system is discussed and the influence of various model parameters is analyzed via numerical simulations. This paper is organized as follows. In Section 2, we describe the model and the computation method of OU stochastic process. In Section 3, we discuss the effects of colored noise by Fokker–Planck equation. Then, in Section 4, the effects of OU noise on the nonlinear system are explicitly experimented. The paper is concluded in Section 5.

2. The bistable potential energy based memory function

To test the LSR induced memory function in the presence of colored noise, we consider a standard overdamped symmetric bistable dynamics, the Langevin-type equation of which is given by

$$\gamma\dot{x} = -\dot{U}(x) + I(t) + y(t), \tag{1}$$

where $\dot{x}$ and $U'(x)$ denote the derivation with respect to t and x, respectively. γ is the dissipation constant. With no loss of generality, we take $\gamma = 1$. $U(x) = -\frac{1}{2}ax^2 + \frac{1}{4}bx^4$ is the reflection-symmetric quartic potential, while the two wells refer to the product states of the reaction, a and b are the parameters of potential. In the absence of input signal, the minima are located at $\pm x_m$, where $x_m = (a/b)^{1/2}$. These are separated by a potential barrier with the height given by $\Delta U = a^2/4b$. Then it can be seen that, with increasing a or decreasing b, the potential well becomes deeper. $I(t)$ is a low amplitude input. The noise $y(t)$ is an OU stochastic process driven by Gaussian white noise with zero mean and delta correlation

$$\dot{y} = -\frac{y}{t_c} + \frac{\sqrt{2D}}{t_c}\xi(t). \tag{2}$$

Where $\xi(t)$ denotes a zero-mean, Gaussian white noise with autocorrelation function $\langle\xi(t)\xi(0)\rangle = 2D\delta(t)$, and D is the strength of the noise. The OU noise $y(t)$ therefore possesses the correlation

$$\big\langle y(t)y(s)\big\rangle = \frac{D}{t_c}\exp\big(-|t-s|/t_c\big), \tag{3}$$

which does approach the case of white noise as correlation time $t_c \to 0$.

The system is driven by a low amplitude input signal $I(t) = I_1(t) + I_2(t)$, with I_1 and I_2 encoding two logic inputs (0 or 1). The two logic inputs give rise to 4 distinct input sets: $(0,0)$, $(0,1)$, $(1,0)$ and $(1,1)$. Since the input sets $(0,1)$ and $(1,0)$ give rise to the same I, the number of distinct logic input sets reduces to 3. But according to the latch truth table (Table 1), the response of Set–Reset latch to inputs sets $(0,1)$ and $(1,0)$ is different unlike the OR/AND logic gates. In order to distinguish between $(0,1)$ and $(1,0)$ states, we modify the encoding of input values as Ref. [19]. The first input I_1 takes the value $-I$ when the logic input is 0 and I when the logic input is 1, while the second input I_2 takes the value I when the logic input is 0 and $-I$ when the logic input is 1, where I is the low input amplitude. We can apply a NOT operation to the second input I_2. The schematic diagram is shown in Fig. 1. Then, corresponding to the 4 sets of original binary inputs (I_1, I_2): $(0,0)$, $(0,1)$, $(1,0)$, $(1,1)$, the consequential input signal sets takes the value $(0,1)$, $(0,0)$, $(1,1)$ and $(1,0)$. We also stress that the former binary inputs sets of $(1,1)$ is a restricted set and does not occur in the latch truth table. So we are left with three input sets, each giving rise to distinct value of I_{in}: 0, $-2I$ and $2I$.

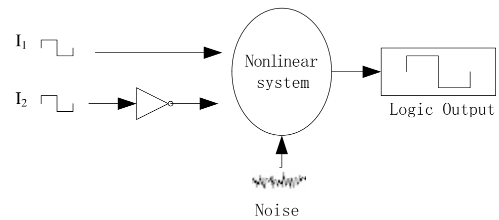

Fig. 1. Schematic diagram of the latch comprised by the nonlinear system.

Logic response of output can be obtained, as in logic operations, by defining a threshold value x^*. If $x_+ > x^*$, i.e., when the system is in the potential well x_+, then the logic output is taken to be 1, and 0 vice versa. Thus, the logic output toggles as the state of the system switches from one well to another.

3. Effects of colored noise

The approximate Fokker–Planck equation for the system (1) without input signal can be obtained by decoupling approximation [20,21]

$$\frac{\partial}{\partial t}P(x,t) = -\frac{\partial}{\partial x}f(x)P(x,t) + \frac{D}{1 - t_c\langle f'(x)\rangle}\cdot\frac{\partial^2}{\partial x^2}P(x,t), \tag{4}$$

where $f(x) = -U'(x)$. The steady state distribution is

$$P_s(x) = N\exp\big(-U_{\mathrm{ou}}(x)\big), \tag{5}$$

where N is the normalization constant, while the potential $U_{\mathrm{ou}}(x)$ takes the form

$$U_{\mathrm{ou}}(x) = \frac{(1 - t_c\langle f'(x)\rangle)U(x)}{D}. \tag{6}$$

Similarly, when the input I is 0.8, the approximate Fokker–Planck equation is

$$\frac{\partial}{\partial t}P(x,t) = -\frac{\partial}{\partial x}g(x)P(x,t) + \frac{D}{1 - t_c\langle g'(x)\rangle}\cdot\frac{\partial^2}{\partial x^2}P(x,t), \tag{7}$$

where $g(x) = -U'(x) \pm 1.6$, and the steady state distribution is

$$P_s(x) = N\exp\big(-U_{\mathrm{ou}}(x)\big), \tag{8}$$

and

$$U_{\mathrm{ou}}(x) = \frac{(1 - t_c\langle g'(x)\rangle)(U(x) \pm 1.6x)}{D}. \tag{9}$$

Fig. 2 shows the responses of the system (1) to Gaussian white noise and exponentially correlated noise. For fixed potential parameters and noise intensity D, the system may behave as a Set–Reset latch, when it is interfered by OU noise, and the outcome coincides with the latch truth table (Table 1). It can be seen that the logical output of Gaussian white noise driven system is wrong in the 2nd, 3rd, 5th, 7th, 11th and 12th time period, while that of exponentially correlated noise driven system is 100% accurate

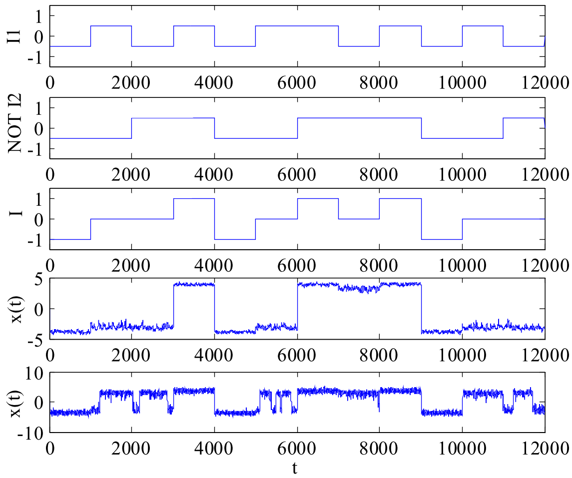

Fig. 2. From top to bottom: panels 1–3 show streams of input I_1, NOT I_2, and $I = I_1 - I_2$; panel 4 shows the response of the system to OU noise with correlation time $t_c = 100$ ms; panel 5 shows the response to Gaussian white noise. With no loss of generality, the value of the two inputs I_1 and I_2 is taken as 0.5 V when the logic input is 1, and −0.5 V when the logic input is 0, other parameters value $a = 0.5$, $b = 0.05$ and $D = 0.8$.

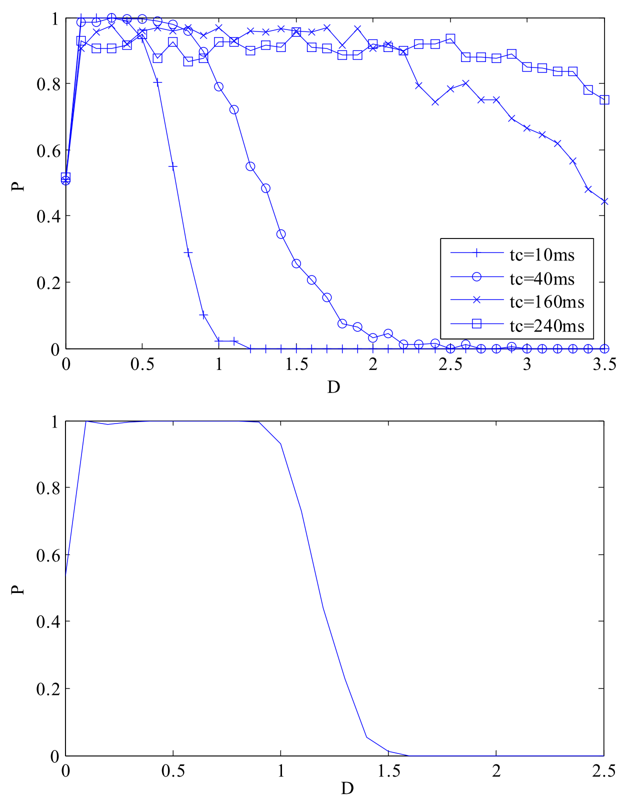

Fig. 3. In the Set–Reset latch, the success probability P(logic) versus noise intensity D is shown driven by exponentially correlated noise at the four fixed correlation time t_c (a), and Gaussian white noise (b). The potential parameters value $a = 0.4$, $b = 0.01$.

with the same system parameters. The details of how the correlation time affecting the response of the system are discussed below.

4. Explicit experiment

We first study the response of the system by changing the noise intensity D. For the fixed potential parameters $a = 0.4$, $b = 0.01$,

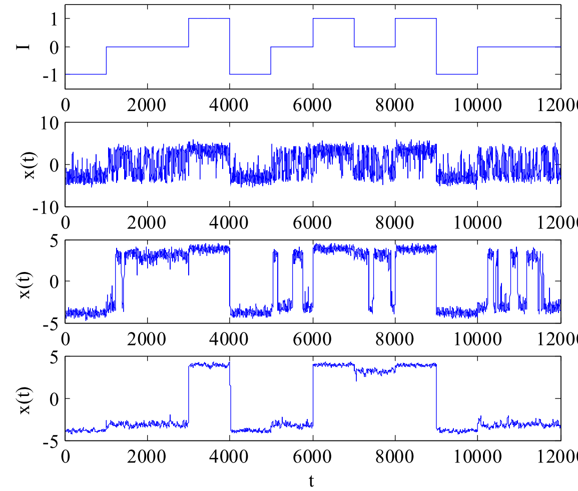

Fig. 4. From top to bottom: panels 1 shows stream of $I = I_1 - I_2$; panels 2–4 show the response of the system to OU noise with correlation time $t_c = 10$ ms, 40 ms, 160 ms, respectively. With no loss of generality, the value of the two inputs I_1 and I_2 is taken as 0.5 V when the logic input is 1, and −0.5 V when the logic input is 0, other parameters value $a = 0.5$, $b = 0.05$ and $D = 0.8$.

the success probability P(logic) as a function of noise intensity D is plotted at four different correlation time t_c (Fig. 3a). In order to compare with the performance of the system under Gaussian white noise, we also plot the response to Gaussian white noise at the same potential parameters (Fig. 3b). The figures show that the success probability P(logic) evolves non-monotonically as the noise intensity D increases, in the presence of exponentially correlated noise. With small correlation time t_c, such as $t_c = 10$ ms and $t_c = 40$ ms, the logical outputs of the system can still be almost 100% accurate in a reasonably wide range of noise strength. As the correlation time t_c increases, the peak performance becomes a bit lower and the optimal noise band shifts to higher levels.

It should be pointed out that with the strongly correlated noise, although the peak performance degrades from 100%, the LSR behavior still exists reliably. The average of the plateau of performance is higher than 95%, which is far better than success probability of coexisting with noise strategy, proposed by Kish [3]. In order to get the highly robust logical output, additional error correction circuits may by applied.

On the other hand, compared with delta correlated noise, the optimal noise band moves to higher D with increasing t_c. In this sense, in strong noisy background, exponentially correlated OU noise may induce the nonlinear system performing as logical function better than delta correlated noise. For instance, with the fixed noise intensity $D = 1.5$, the success probability of getting the correct logical output is only 1% driven by delta correlated noise, while the success probability is 30%, 90%, 97% respect to $t_c = 40$ ms, $t_c = 160$ ms, $t_c = 240$ ms, separately.

For fixed noise intensity D, typical output streams of the system are plotted at three different correlation time t_c (Fig. 4). Fig. 4 shows at strong noise intensity, for instance $D = 0.8$, stronger correlated noise ($t_c = 160$ ms) may induce the nonlinear system to obtain robust logical output, while responses of the system driven by the weak correlated noise occur error more or less.

Fig. 5 displays the success probability P(logic) versus correlation time t_c at fixed noise intensity D. It can be seen that the success probability P(logic) evolves non-monotonically, first increasing and then decreasing as the correlation time t_c increases. This is a celebrated resonance-like effect. The result indicates that under some circumstances the reliability of the colored noise driven nonlinear system can be improved by adjusting the correlation time t_c. The figure also shows there is a wide range of intermediate t_c in

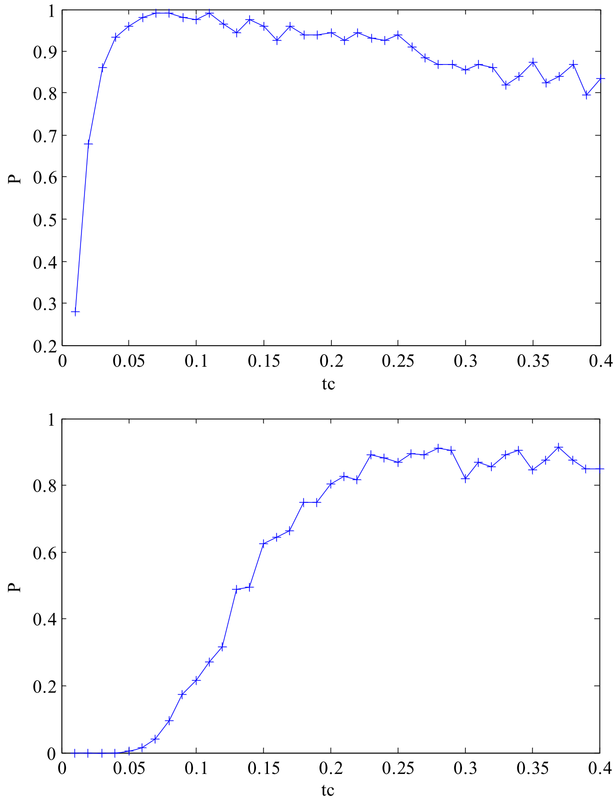

Fig. 5. In the Set–Reset gate, the success probability $P(\text{logic})$ versus correlation time t_c is shown at the fixed noise intensity $D = 0.8$ (a) and $D = 3.0$ (b). The potential parameters value $a = 0.4$, $b = 0.01$.

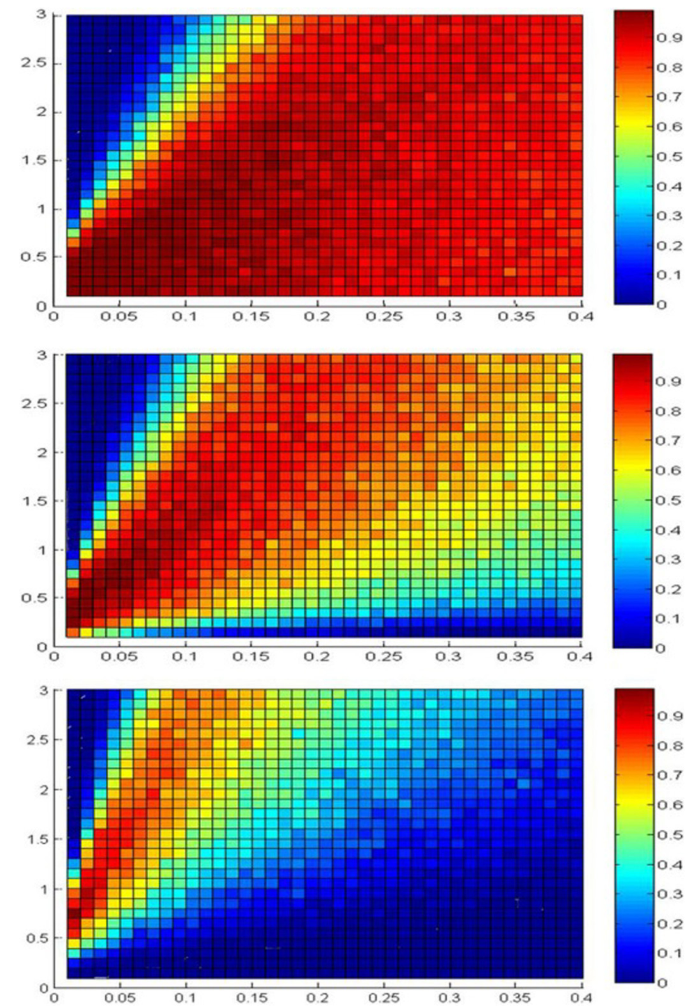

Fig. 6. In a 30×40 lattice, for the Set–Reset latch operation, the success probability $P(\text{logic})$ versus the correlation time t_c (x axis) and noise intensity D (y axis) is drawn at three different sets of potential parameters, $a = 0.4$, $b = 0.01$; $a = 0.5$, $b = 0.015$; $a = 0.5$, $b = 0.05$, respectively, from top to bottom.

which the logic response of the system can be obtained reliably in the strong noise background. For instance, for the fixed noise intensity $D = 3.0$, the success probability can reach almost 90% in a considerable wide range of noise intensity.

For Set–Reset latch gate, the success probability $P(\text{logic})$ as a function of correlation time t_c and noise intensity D is plotted in Fig. 6, for the three different sets of potential parameters. The figure shows that the optimal band of noise intensity as well as correlation time shift to higher levels with the increasing of the potential well depth. Above $D \sim 0.5$, the figure shows that the success probability $P(\text{logic})$ can evolve non-monotonically as the t_c increases. The LSR effect as a result of t_c still exists. However, for small noise intensity D, e.g., below $D \sim 0.5$, it can be seen that the success probability $P(\text{logic})$ decreases monotonically as the t_c increases. The LSR phenomenon induced by changing t_c disappears. The result shows that the LSR induced by changing t_c only can be obtained at large enough noise intensities. The outcome is consistent with the study of Ref. [10]. Besides, Fig. 5 illustrates that the response of the nonlinear system subjected to strong correlated noise is more sensitive to the changing of the shape of the potential energy well than that subjected to weak correlated time. So in order to get robust logical function, the potential energy well parameters should be picked carefully.

5. Conclusion

In this work, we have demonstrated that a symmetric bistable nonlinear system can perform as memory device, specifically yielding Set–Reset latch operation, subjected to OU noise, driven by two low amplitude and streaming in any random sequence inputs. We take explicit numerical experiments to study the effects of the exponentially correlated noise on the outcome of the nonlinear system. The LSR phenomenon is obtained, and the reliability of the logic system can evolve non-monotonically as a function of noise intensity. Driven by weak correlated noise, the logic outcome of the system is similar to that driven by Gaussian white noise. The peak performance can yield 100% accuracy at certain intermediate noise. On the other hand, although the peak performance of the system degrades from 100% accuracy subjected to strong correlated noise, the optimal band of noise intensity shifts to higher level and becomes wider. This means increasing correlation time tends to refine the robustness of the logic system in strong noisy background. Meanwhile, we point out that the success probability of getting the right logic outcomes at the finite correlation time can be less affected by the shape of the potential energy well than that at large correlation time.

With the future miniaturization of microelectronics, the thermal noise has threatened to end the Moore's law. As latch is the fundamental building block of a computing machine and is omnipresent in computers and communication systems, proposal of the novel implementation of latch which can behave robustly at strong noisy background has far-reaching consequences.

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